MOTION IN TWO DIMENSIONS

PROJECTILE MOTION

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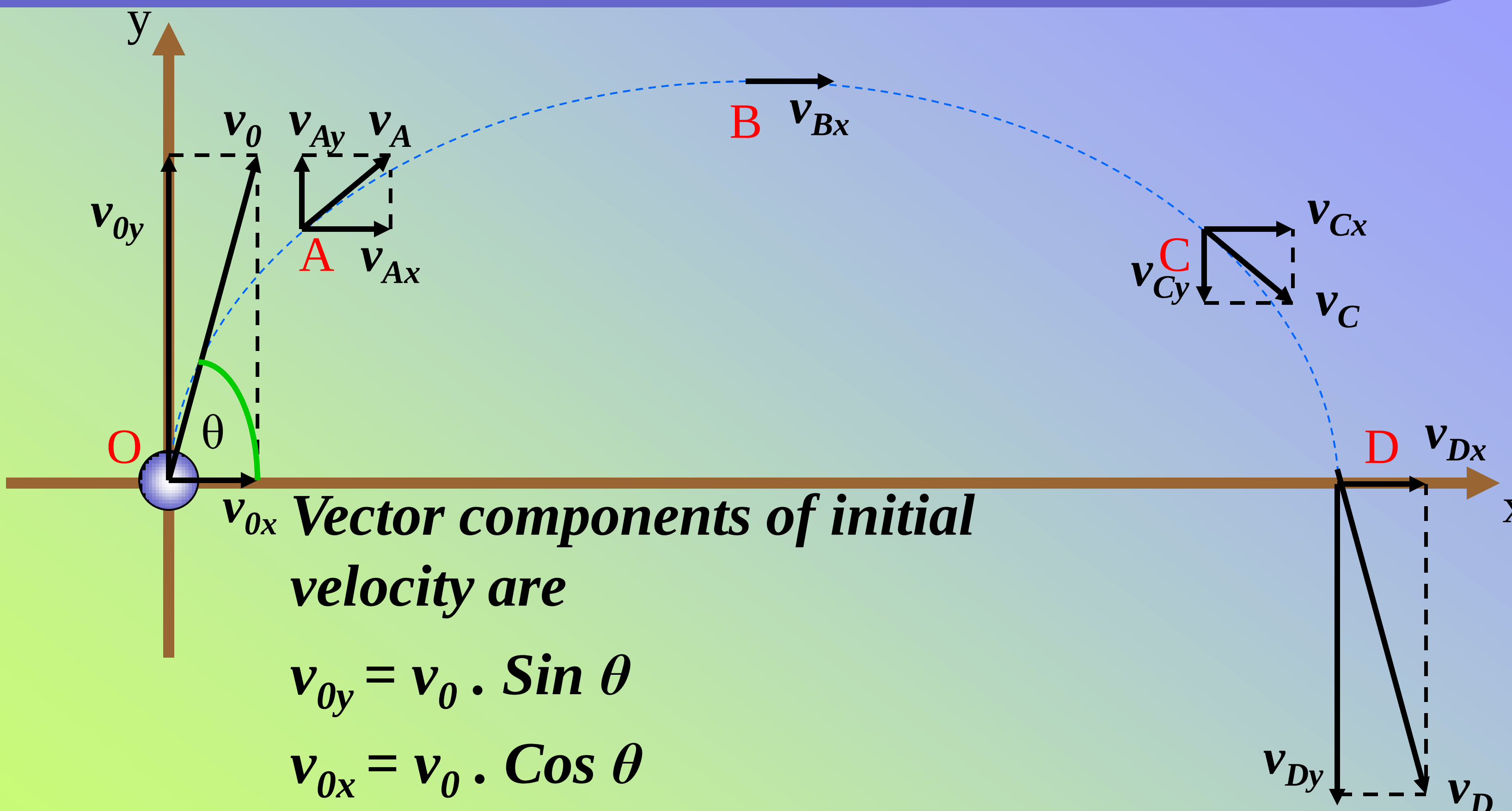
CONTENTS :

- Basic Concepts
- Physics and Mathematical Analysis
- Problem

Projectile Motion/ Parabolic Motion

- Projectile motion is a motion in two dimensions
- Because in x axis there is no acceleration, the motion in x axis is a uniform velocity motion
- The acceleration in y axis is acceleration of gravity
- It means that motion in y axis is an accelerated motion

PROJECTILE MOTION



Vector components of initial velocity are

$$v_{0y} = v_0 \,.\, \sin\theta$$

$$v_{0x} = v_0 \,.\, \cos\theta$$

$$v_{0x} = v_{Ax} = v_{Bx} = v_{Cx} = v_{Dx}$$

Projectile Motion

Remember that in x axis the motion is in uniform velocity and in y axis there is accelerated motion

For uniform velocity motion we have:

$x = v\ \ t$

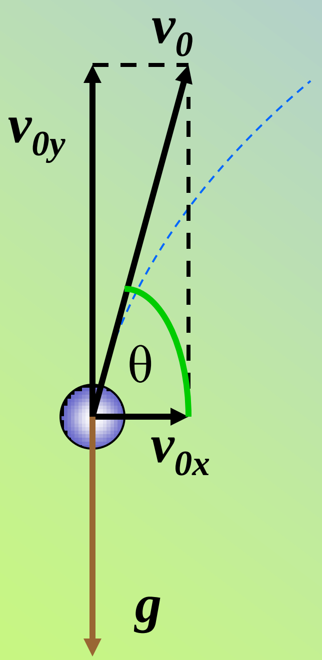
For accelerated motion we have :

$s = v_0 t + \frac{1}{2} at^2$

$v_t = v_0 + at$

$v_t^2 = v_0^2 + 2as$

PROJECTILE MOTION



Horizontal motion (Uniform Velocity)

$$v_x = v_{0x} = v_0 \,.\, \cos\theta$$

$$x = v_{0x} \,.\, t = v_0 \,.\, \cos\theta \,.\, t$$

Vertical motion (Accelerated Motion)

Let upward be positive direction, so that the value of g is negative

$$v_y = v_{0y} - g.t = v_0 \,.\, \sin\theta - g.t$$

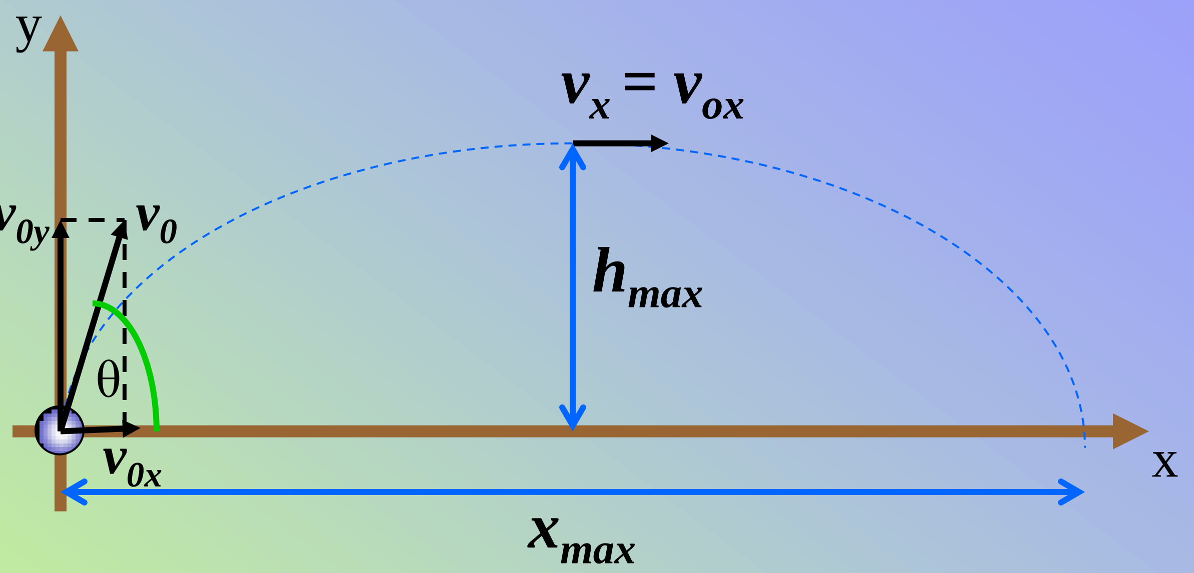
$$y = v_{0y}.t - \tfrac{1}{2}.g.t^2 = v_0 \,.\, \sin\theta.\, t - \tfrac{1}{2}.g.t^2$$

Projectile Motion

From those equation we know that :

- When the projectile goes up, its velocity in y axis is decreasing, zero at the highest point and then getting faster when it goes down
- The velocity of the projectile in x axis remains constant because there is no acceleration in x axis

Determine the highest point



$$v_y = 0$$

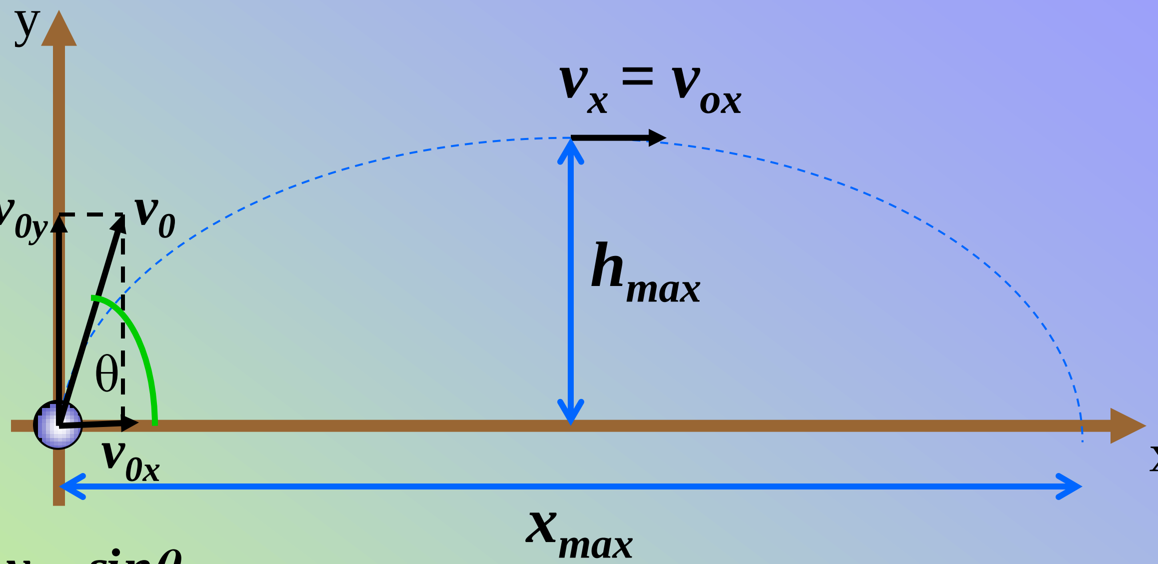
$$0 = v_0 \cdot sin\theta - g \cdot t$$

$$t = \frac{v_0 \cdot sin\theta}{g}$$

$$y_{max} = h_{max} = v_0 \cdot sin\theta \cdot \frac{v_0 \cdot sin\theta}{g} - \frac{1}{2} \cdot g \cdot \left(\frac{v_0 \cdot sin\theta}{g}\right)^2$$

$$h_{max} = \frac{v_0^{\ 2} \cdot sin^2\theta}{2 \cdot g}$$

Determine horizontal range



$$t_{hmax} = 2 \cdot t_{ymax} = \frac{2 \cdot v_0 \cdot sin\theta}{g}$$

$$x_{max} = v_0 \cdot cos\theta \cdot \frac{2 \cdot v_0 \cdot sin\theta}{g} = \frac{2 \cdot v_0^{\,2} \cdot sin\theta \cdot cos\theta}{g}$$

$$x_{max} = \frac{v_0^{\,2}}{g} \cdot sin2\theta$$

Problem :

The launching speed of a certain projectile is five times the speed that it has at its maximum height. Calculate the elevation angle at launching!

Click here for the answer

ANSWER

Note that at the highest point, vertical velocity of the projectile is zero. So the velocity at the highest point is equal to v_x.

$$v_0 = 5v_x$$

$$v_0 = 5v_0 \cos\theta$$

then we can get the value of θ

$\theta = \cos^{-1}(1/5)$